SPS Momentum Calibration and Stability in 2003

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Abstract

The stability of the SPS beam momentum is important to ensure good beam quality for the LHC. Field measurements in the SPS reference magnet have been used to determine the stability of the SPS field over the duration of a run. Central frequency measurements were performed at 26, 370 and 400 GeV/c to study systematic effects of the SPS sextupole alignment discovered in 2002 during a precise energy calibration. A clear momentum dependance of the systematic shifts is observed.
1 Introduction

At the end of the 2002 SPS run the beam momentum at the LHC extraction energy of 450 GeV/c was calibrated to obtain a precise knowledge of the SPS beam parameters for the LHC commissioning [1]. The calibration method was based on revolution frequency measurements of proton and Pb$^{53+}$ beams injected into the same magnetic machine. The measurement was successful, but it also revealed an unexpected radial alignment error of 0.7 mm between the LSF and LSD sextupole families.

The 2003 SPS run was used to study this effect in more detail, with similar measurements of central RF frequency at 26, 370 and 400 GeV/c. In the context of injection into the LHC, NMR measurements were used to evaluate the momentum stability of the SPS beam over an entire run.

This note presents the results of the field stability measurements. The energy calibration principle is then briefly recalled and the full set of new central frequency measurements is presented and analyzed.

2 Magnetic Field Measurements

Magnetic field measurements are available at the SPS from the reference dipole magnets installed in BA3. The field can be measured either through a set of NMR probes covering fields ranging between 0.05 and 2.6 T, or by the BTRAIN instrument. The BTRAIN field measurement device is based on an induction coil installed in the reference magnet. A voltage-to-frequency converter is used to generate a pulse train, each pulse corresponding to a field increment of 0.1 Gauss. The BTRAIN data can be acquired through the standard SPS RF application, while the synchronization with the SPS magnetic cycle is programmed over a set of RF system TG3 modules.

2.1 NMR Measurements

The NMR probes provide field measurements with a relative accuracy of $10^{-6}$ if the field is sufficiently uniform and stable in time. At the SPS measurements on the flat tops of flat bottoms are delicate because of the fast cycle times. As a consequence the NMR resonance must be tuned in manually with the help of an oscilloscope and the relative accuracy is limited to to $\approx 10^{-4}$. In 2003 measurements have been performed regularly to provide references and determine the field stability, in particular for the LHC beam cycle. In all cases the stability of the field is at the level of $10^{-4}$ or better over the entire duration of the run. The stability of the measurement at 450 GeV/c for the LHC cycle is shown in Fig. 1. The average field settings corresponding to some of the 2003 SPS settings is given in Table 1. The field measurements can be used to determine the expected beam momentum using the calibration performed at 450 GeV/c in 2002 [1]. The momentum $P_b$ is estimated through simple scaling

$$P_b = \frac{B}{B_{ref}} P_{ref}$$

where $P_{ref} = 449.16 \pm 0.14$ (GeV/c) and $B_{ref} = 2.0251 \pm 0.0002$ (T) are the reference momentum and magnetic field from the momentum calibration. In general the extrapolated momentum
Table 1: NMR field measurements of the SPS reference magnet for a number of 2003 SPS operational cycles. The first column indicates the nominal reference momentum. The third column gives the average dipole field $\bar{B}$ over the run and the fourth column the typical accuracy $\sigma_B$ of an individual NMR measurement. The last column is estimated the beam momentum $P_b$ obtained by extrapolation of the momentum calibration at 450 GeV/c [1].

lies below the 'nominal' momentum. It is important to observe that such a simple scaling is only valid when there are no saturation effects at the edge of the main dipoles. End field saturation generates non-linearities in the relation between the central field that is measured and the integrated magnetic field that defines the beam momentum.

Figure 1: Relative dipole field variation over the SPS run in 2003 at 450 GeV/c for the LHC beam cycle. The measurements have been performed with a NMR probe. The accuracy of the individual measurements is limited to $(1-2) \times 10^{-4}$ because of the short duration of the flat top (1 second). The field is stable over the run within the measurement accuracy.
Figure 2: Top: the magnetic field evolution during the LHC beam cycle (540) measured with the BTRAIN (blue points) is compared to the reference function (red curve). The field is given relative to its injection value. Bottom: Difference between the BTRAIN measurements and the reference field function in mT as a function of the time in the cycle.
Figure 3: Top: the nominal derivative of the SPS main dipole field $\dot{B}$ (red) is compared to the derivative measured with the BTRAIN along the LHC cycle ramp (blue points). The same measurement is shown for the Indium cycle in the bottom plot. The time shift of 10-20 ms between the measurements (blue points) and the reference (red curve) may be due to a combination of effects (eddy currents and instrumental). Contrary to the case of the LHC cycle, there is no significant deviation of $\dot{B}$ from the reference for the Indium cycle.
2.2 BTRAIN Measurements within Cycles

The BTRAIN measurements can be used to map the field changes over an entire SPS cycle with a very high granularity in field (0.1 G) and time (1 ms). Differences with respect to the expect field evolution can easily be observed with this instrument. An example is shown in Fig. 2 for the standard LHC cycle (SC540). Field differences of up to 6 mT corresponding to relative momentum errors of more than $3 \times 10^{-3}$ are visible in the second half of the ramp. The field derivative $\dot{B} = dB/dt$ as reconstructed from the same data is shown in Fig. 3. For fields above 1 T, $\dot{B}$ no longer follows the expected derivative, an effect that was already seen in RF studies [2]. The same effect is not visible on SC400 that was used in 2003 for the Indium programme.

For the LHC cycle 540, the integrated field change between injection and flat top as measured by the NMR probes is 1.9084 T, while the BTRAIN measurements yield a value of 1.9148 T, i.e. the two devices differ by 6 mT. The difference may be explained by a longitudinal inhomogeneity of the dipoles or by the lower accuracy of the BTRAIN device compared to the NMR probes. On the 1 second long 450 GeV/c flat top, the BTRAIN indicates a field drift of $10^{-4}$ T.

3 Central Frequency Measurements

A precise absolute momentum calibration of beams can be obtained from a comparison of revolution frequencies for two ion species of different charge over mass ratio. Using this technique, a precise energy calibration was performed at LEP with protons and positrons at 20 GeV/c [3]. In 2002 the same method was used successfully to calibrate the SPS momentum at 450 GeV/c using proton and Pb$^{53+}$ beams [1]. We briefly recall the main points of the energy calibration procedure.

The speed $\beta c$ of a particle, where $c$ is the speed of light, can be related to the revolution frequency $f_{rev}$ and the RF frequency $f_{RF}$ through

$$\beta c = C f_{rev} = \frac{C f_{RF}}{h}$$

where $h$ is the harmonic number of the RF system, $h = 4620$ for the SPS. $C$ is the machine circumference. To determine $\beta$ and therefore the particle momentum, both the machine circumference and the revolution frequency must be known. To determine those parameters at the same time, the revolution frequency is measured for two particles with different charge over mass ratio that are injected into exactly the same magnetic machine and on the same orbits.

For the SPS and the LHC, the speed $\beta p c$ of the proton beam is related to its momentum $P$ and its rest mass $m_p$ by

$$\beta_p^2 = \frac{P^2}{P^2 + (m_p c)^2}.$$  \hspace{1cm} (3)

An ion with charge $Ze$, injected into the same magnetic machine and on the same orbit than the proton beam has a momentum $P_i = ZP$. The proton equivalent momentum of the ion is defined by $P = P_i/Z$. The speed $\beta_i c$ of the ions is

$$\beta_i^2 = \frac{P^2}{P^2 + (m_i c/Z)^2}.$$  \hspace{1cm} (4)
with $m_i$ the ion rest mass. The two equations for $\beta_p$ and $\beta_i$ can be solved for the proton beam momentum $P$, yielding

$$P = m_p c \sqrt{\frac{\kappa^2 \mu^2 - 1}{1 - \kappa^2}}$$

(5)

with

$$\kappa = \frac{\beta_i}{\beta_p} = \frac{f_{iRF}}{f_{pRF}}$$

(6)

and

$$\mu = \frac{m_i}{Z m_p}.$$  

(7)

Equation 5 can be approximated by

$$P \approx m_p c \sqrt{\frac{f_{pRF}^2}{2\Delta f} (\mu^2 - 1)}$$

(8)

where $\Delta f = f_{pRF}^i - f_{pRF}^h$ is the RF frequency difference between the beams.

### 3.1 Central Frequency Measurement Principle

In practice the central frequency is determined by centering the beams in the machine sextupoles where the betatron tune is independent of the radial position (i.e. RF frequency). To determine the magnetic center of the sextupoles, the tune is measured as a function of the radial position for a number of different chromaticity settings (positive and negative). As long as the RF frequency changes are performed within a short range around the central frequency, the tune dependence on RF frequency is linear for a given chromaticity setting. The central frequency corresponds to the crossing point of lines for different chromaticity, as can be seen in Figures 4 and 5. The measurement can be performed by varying either the horizontal or the vertical chromaticity, in which case the beams are mainly sensitive to the horizontal and vertical sextupole families.

At the SPS, the chromaticity is corrected using 108 lattice sextupoles, 54 LSD type (vertical focusing) and 54 LSF type (horizontal focusing) magnets. The LSD magnets are grouped in 2 families, LSDA (18 magnets) and LSDB (36 magnets). The LSF are grouped in 3 families, LSFA (24 magnets), LSFB (18 magnets) and LSFC (12 magnets). For the super-cycles and optics considered for this note, the strengths of LSFA and LSFC families are identical. Since horizontal chromaticity changes are mainly performed using the LSF sextupoles, while the vertical chromaticity is mainly varied using the LSD family, the central frequencies obtained from the two planes are determined by the alignment of those two families. Differences in central frequency between horizontal and vertical planes are directly related to alignment errors between families.

### 3.2 Energy Calibration in 2002

The calibration of the SPS beam momentum at 450 GeV/c in 2002 relied on beams of protons and $\text{Pb}^{53+}$, the later being used instead of standard $\text{Pb}^{82+}$ for its larger value of $\mu$ and therefore larger $\Delta f$ (Equation 8). A very accurate calibration was obtained, but the results given in Table 2 are limited by unexpected systematic effects on the machine circumference. The measurements shown in Fig. 4 indicate a different central RF frequency (and therefore machine circumference) for the horizontal and vertical sextupole families since the crossing points of the
Figure 4: Tune dependence on RF frequency for different settings of the machine chromaticity for proton (top) and Pb$^{53+}$ beams (bottom) at a proton equivalent momentum of 450 GeV/c (from Reference [1]). This measurement was performed in 2002. For each beam two measurement are obtained by varying the horizontal and the vertical chromaticity and observing the corresponding tune. The central RF frequency (and its error) that corresponds to each crossing point is indicated for the horizontal (in blue, $Q \simeq 0.18$) and vertical (in red, $Q \simeq 0.14$) plane.
Figure 5: Tune dependence on RF frequency for different settings of the machine chromaticity for protons at 400 GeV/c (top) and In$^{49+}$ at a proton equivalent momentum of 370 GeV/c (bottom). The central RF frequency (and its error) that corresponds to a crossing points is indicated for each measurement set. For each beam two measurement are obtained by varying the horizontal (blue) and the vertical (red) chromaticity.
Table 2: Beam parameters obtained from the SPS momentum calibration in 2002, averaged over the data obtained from the horizontal and vertical planes. The errors are determined from the systematic differences between the data of the two planes.

Chromaticity curves for the two planes do not correspond to the same RF frequency. This effect is visible both for proton and for Pb\textsuperscript{53+} beams.

### 3.3 Central Frequency Measurements in 2003

To understand the observed difference of central frequencies, additional measurements were performed in 2003 for protons beams of 26 and 400 GeV/c as well as for In\textsuperscript{49+} beams at a proton equivalent momentum of 370.1 GeV/c. The high energy measurements are shown in Fig. 5: they are all of excellent quality, with error on the central frequency of \( \sim 1 \) Hz. The systematic difference between the planes is present in all high energy data, but the absolute value is decreasing in parallel with the beam momentum. At 26 GeV/c the measurements were performed in detail by varying the chromaticity with each of the 5 SPS sextupole families independently. The results shown in Fig. 6 for the different measurement sets do not exhibit a significant dependence on the sextupole family. The measurement accuracy is however much lower at 26 GeV/c due to important collective effects. A summary of all measurements is given in Table 3.

The central frequency measurements can be used to determine the absolute SPS momentum at the corresponding energies using Equations 2 and 3. The calibration performed in 2002 at 450 GeV/c is used as reference point. The results, labelled \( P_{frf} \), are given in Table 4 together with the beam momentum \( P_b \) estimated from the reference magnet field measurements. At 370 and 400 GeV/c \( P_{frf} \) has a poor accuracy due to the large systematic effect between horizontal and vertical sextupole families. At 26 GeV/c the result for \( P_{frf} \) is more competitive and accurate than \( P_b \) because of the significantly lower \( \gamma \) of the proton beams, since the momentum error \( \sigma_P \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Proton momentum ( P ) (GeV/c)</td>
<td>449.155 ± 0.136</td>
</tr>
<tr>
<td>Reference field ( B ) (T)</td>
<td>2.0251 ± 0.0002</td>
</tr>
<tr>
<td>Central orbit length ( C ) (m)</td>
<td>6911.5662 ± 0.0024</td>
</tr>
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</table>

Table 3: Summary table of all central frequency measurements. All frequencies are corrected for tidal effects [1].
Figure 6: Summary of central frequency measurements at 26 GeV/c, displayed as a function of the sextupole family/combination that is used for each measurement. The measurements labelled ‘Chroma-H’ and ‘Chroma-V’ use the standard matched chromaticity knobs. The bands corresponds to the $\pm 1 \sigma$ errors for the combined result of each plane. The measurements have been weighted according to the errors and the number of sextupoles involved in the measurement set.

Figure 7: Momentum dependence of the difference between the central RF frequency measured with the LSD and LSF sextupole families. The color are used to distinguish measurements performed with the different beam types.
<table>
<thead>
<tr>
<th>Type</th>
<th>Setting (GeV/c)</th>
<th>Beam Momentum (GeV/c)</th>
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<tr>
<td>p</td>
<td>26</td>
<td>26.02 ± 0.01</td>
</tr>
<tr>
<td>In$^{49+}$</td>
<td>370</td>
<td>368.4 ± 2.3</td>
</tr>
<tr>
<td>p</td>
<td>400</td>
<td>398.0 ± 14.0</td>
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<tr>
<td>p</td>
<td>450</td>
<td>449.16 ± 0.14</td>
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<th>Type</th>
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<th>Momentum (GeV/c)</th>
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<td>In$^{49+}$</td>
<td>370</td>
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<td>p</td>
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<td>398.22 ± 0.13</td>
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<tr>
<td>p</td>
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Table 4: Summary of momentum calibrations obtained from central RF frequency ($P_{frf}$) and field measurements with the NMR ($P_b$). The 450 GeV/c calibration of 2002 is the reference point. The momentum $P_{frf}$ is obtained from the average central frequency, while the difference between planes is used to define the error. For high fields, the NMRs provide more accurate measurements because $P_{frf}$ is limited by the central frequency systematic effects. At 26 GeV/c $P_{frf}$ is again competitive and more accurate because of the lower $\gamma$ of the protons. The momentum $P_b$ obtained from the NMR data at 26 GeV/c is mostly likely wrong.

is related to the error $\sigma_\beta$ on the speed and $\sigma_f$ on the RF frequency by

$$\frac{\sigma_P}{P} = \gamma^2 \frac{\sigma_\beta}{\beta} \propto \gamma^2 \frac{\sigma_f}{f_{RF}}.$$  \hspace{1cm} (9)

A significant difference is observed with respect to the momentum extracted from the NMR data. The later is mostly likely biased because the simple extrapolation based on the central magnetic field does not hold over the large span from 450 to 26 GeV/c.

The dependence of the central RF frequency difference between vertical and horizontal sextupoles is shown Fig. 7 as a function of the beam momentum. At 26 GeV/c the shift between LSF and LSD families is not statistically significant, in particular if one considers that effects of 10-20 Hz can be due to the closed orbit r.m.s. of 1-2 mm that are typical for the SPS.

The apparent momentum dependence of the central frequency difference between the two planes may be due to magnetic forces that displace the sextupoles on their girders by ±0.3 mm, with opposite sign for the 2 families, although this seems to be a rather large displacement. Another explanation is a shift of the magnetic center of the sextupoles due to interferences with stray fields from the nearby dipoles or quadrupoles.

4 Conclusion and Discussion

The momentum stability of the SPS beam at high energy was confirmed to be excellent, around or better than $10^{-4}$. The stability should be sufficient to ensure a high capture efficiency in the LHC with the 400 MHz LHC RF system.

The central frequency was measured at SPS beam momenta of 26, 370, 400 and 450 GeV/c. The measurements confirmed a systematic radial shift of the magnetic center of the LSD and LSF sextupole families in the SPS. The shift depends strongly on the beam momentum. There are a number of ‘explanations’ for this dependance of the magnetic center on the field itself. One possibility is that the magnets are displaced mechanically by magnetic forces. Another possibility is a superposition of stay fields from the SPS dipoles and quadrupoles that shift the magnetic center of the sextupoles. Finally it is not excluded that the magnetic center of the
sextupoles itself has a dependence on the field level. Further central frequency measurements are foreseen in 2004 at 270 GeV/c. Direct measurements of sextupole movements during the SPS cycle can be envisaged with the help of the survey group during the next cold-checkout of the SPS.

5 Acknowledgements

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References

