The speed of the particles βc, where c is the speed of light, can be related to the revolution frequency f_{rev} and the corresponding RF frequency f_{RF} through

$$\beta c = C f_{rev} = \frac{C f_{RF}}{h} \tag{1}$$

where h is the harmonic number of the RF system. C is the machine circumference.

The speed $\beta_p c$ of the proton beam is related to its momentum P, the main parameter of interest, and its rest mass m_p by

$$\beta_p^2 = \frac{P^2}{P^2 + (m_p c)^2} \; . \tag{2}$$

The variation of β as a function of *mom* is

$$\frac{d\beta}{\beta} = (1 - \beta^2)\frac{dP}{P} = \frac{1}{\gamma^2}\frac{dP}{P} \tag{3}$$

Consequently we have

$$\frac{df_{RF}}{f_{RF}} = \frac{d\beta}{\beta} = \frac{1}{\gamma^2}\frac{dP}{P} = \frac{1}{\gamma^2}\frac{dB}{B} \tag{4}$$

where B is the magnetic field. This equation is valid at constant radius/circumference and describes how the RF frequency varies with the momentum (or with the field). Clearly the RF frequency tends towards an asymptotic value given by $\beta = 1$ in Eq. 1.

At constant field the momentum depends depends on the RF frequency

$$\frac{dP}{P} = -\frac{1}{\alpha_c - 1/\gamma^2}\frac{df_{RF}}{f_{RF}} \tag{5}$$

where

$$\alpha_c = 1/\gamma_t^2 \tag{6}$$