# **Prototyping LHC orbit control**

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## Abstract

Orbit correction consists in varying the strengths of the corrector magnets to make the measured beam position match a predefined reference. In the two LHC rings, this involves around 1000 beam position monitors and over 500 orbit correctors in each plane. The orbit control loop of the LHC must be able to compensate orbit drifts at frequencies between  $10^{-2}$  and 1 Hz. In this paper we investigate correction schemes and control designs that could be used for the LHC.

# **1 INTRODUCTION**

Orbit control will play an important role in the LHC due to the tight aperture and collimation constraints [1]. The stability of the orbit in critical sections must be better than  $\sigma/5$  ( $\sigma$  = r.m.s. beam size), which corresponds to  $\sim 20$  to 100  $\mu$ m displacements. The exact frequency spectrum of orbit distortions in the LHC is still unknown, but the experience gained at LEP can be used to estimate the effects due to ground motion and other perturbations [3]. While in LEP the horizontal r.m.s. orbit drift rarely exceeded 0.3 mm during a few hour long fill, the vertical movements were faster, with integrated drifts exceeding 4 mm r.m.s. The vertical orbit drifts were mainly driven by movements of the lowbeta quadrupoles. In the frequency range of 1 to 100 Hz, the orbit oscillation amplitudes were limited to 10 microns or less. Similar figures have been reported at HERA [4]. Orbit changes exceeding 3 to 4 mm r.m.s. had also to be corrected at LEP during the ramp and the beta-squeeze.

We expect that orbit movements at the LHC will exhibit similar slow drifts due to ground motion. In addition the LHC will be affected by slow drifts due to the decay of persistent currents in the superconducting magnets [6] that are most important around the 450 GeV injection plateau. Faster orbits changes are expected during the snapback in the early part of the ramp [2] ( $\sim 0.4$  mm r.m.s. over  $\sim 30$  s) and during the squeeze at 7 TeV ( $\sim 5$  mm r.m.s. over  $\sim 100$  s).

In this paper, a general orbit correction strategy is presented that could be used for a global or local feedback loops. The approach is general and could be envisaged for other site-wide feedback applications such as a tune loop.

#### 2 CORRECTION ALGORITHM

Let the vector  $\vec{x}$  (size N) represent the beam position measured at the monitors (BPMs) and the vector  $\vec{y}$  (size M) the corrector strengths. The task of the orbit correction is to determine a set of corrector strengths that minimises the difference between  $\vec{x}$  and a reference orbit  $\vec{x}_{ref}$ :

$$\vec{x} - \vec{x}_{ref} + \mathbf{A}\vec{y} = 0 \tag{1}$$

The response matrix **A** describes the relation between the corrector kicks and the beam position changes at the BPMs. For LHC, **A** is not a square matrix since N > M and Eq. 1 is over-constrained. MICADO and Singular Value Decomposition (SVD) [5] are two common algorithms used to perform the least square minimisation of  $\|\vec{x} - \vec{x}_{ref} + \mathbf{A}\vec{y}\|^2$ . Here we consider only the SVD algorithm that decomposes matrix **A** into a product of three matrices :

$$\mathbf{A} = \mathbf{U}\mathbf{W}\mathbf{V}^T \tag{2}$$

where U and V are orthogonal matrices and  $V^T$  represents the transpose of V. W is a diagonal matrix containing the eigenvalues  $w_i$  that are proportional to the r.m.s. of the orbit response corresponding to its associated eigenvector (the columns of matrix V). The solution of the least square problem is

$$\vec{y} = -\mathbf{A}^{\dagger} \vec{x} = -\mathbf{V} \mathbf{W}^{-1} \mathbf{U}^T \vec{x}$$
(3)

where  $\mathbf{A}^{\dagger}$ , the "pseudo-inverse" of  $\mathbf{A}$ , can be made nonsingular by zeroing all elements of  $w_i$  smaller than a predefined cut off. For large machines such as SPS, LEP or LHC, the eigenvalues  $w_i$  cover 2 to 5 orders of magnitude (Fig. 1). Correction of large-scale orbit patterns is done with large eigenvalues while smaller values correspond to smaller structures such as local bumps. The number of eigenvalues retained determines therefore the spatial resolution of the correction [5].



Figure 1: Spectrum of orbit response eigenvalues for LHC. The eigenvalues with  $\log_{10}(w_i/w_1) < -4$  correspond to near singular solutions around the the interaction regions.

### **3 ARCHITECTURE**

A centralised solution for orbit correction is envisaged where a single processor receives and sends data to equipment front ends distributed around the ring. The actuators of the system are the  $\approx$  1000 orbit corrector magnets and their associated power converters (PCs) that are accessed via a deterministic field-bus. The sensors are BPMs that measure the transverse position in both planes. The present design foresees that each BPM delivers data at 10 Hz. For 500 BPMs per ring, this corresponds to a data stream of  $\approx$  100 kBytes/s per ring.

## **4 DYNAMICS OF THE SYSTEM**

Time constants and delays are important in a feedback loop because they determine performance and robustness of the system. For the LHC orbit feedback loop, the entire dynamics is due to the power converters and associated magnets, the vacuum chamber and the overall time delay.

#### 4.1 Power converters and magnets

The majority of superconducting orbit corrector magnets has an inductance of 7 H and a warm cable resistance of 30 m $\Omega$  giving a natural time constant of 230 s. The magnets are driven by 4-quadrant power converters ( $\pm$  8 V,  $\pm$  60 A) that are equipped with a digital control loop. This loop is using the available power to accelerate the response of the system [8]. In our control loop design, we will therefore use the effective time constant rather than the natural time constant.

At 450 GeV small current steps of  $\pm 0.1$  A, corresponding to 2  $\mu$ rad deflections, are largely sufficient to correct the orbit deviations induced by the snapback. For such small steps, the effective time constant is reduced to 100 ms. During the squeeze and in coast (7 TeV) the effect of the same current step is reduced by a factor 20 compared to injection. If the same deflection is required, the effective time constant will be increased to a few seconds. Alternatively, the maximum kick strength could be reduced significantly. In either case, the result is a reduction of the feedback gain at high energy.

#### 4.2 Vacuum chamber

The magnet response is affected by eddy currents on the vacuum chamber which acts as a low pass filter with characteristic time constants of 12 ms for cold and 2 ms for warm chambers. Note that this is small compared to the time constant of magnet and PC.

#### 4.3 *Time delay*

A delay in a feedback leads to a phase shift which always degrades the stability and reduced the gain of the loop. Within limits, time delays can be compensated for by adjusting the feedback gain and/or by using lead or lag compensation. A first delay arises from data acquisition and transport of the data across the LHC site via computer network. From tests with the CERN SPS network, this delay is estimated to be in the range of 20 to 40 ms.

Another delay is due to the execution of the correction algorithm. Although it largely depends on issues such as processor speed, compiler and operating system, it is likely that it can be kept in the range of 10 to 20 ms even for the largest response matrices.

Finally there is a time delay due to the PC digital controller which has been estimated at 30 to 50 ms.

The overall system delay is therefore expected to be in a range of 50 to 120 ms, and for the design of the loops that follows, we use a conservative value of 100 ms.

### **5** CONTROLLER DESIGN

The present design of a digital controller for the orbit feedback is based on classical control theory using a frequency domain approach and z-transformations. A detailed description of the methods can be found in Ref. [9].

#### 5.1 Sampling time

The sampling time is the time between two consecutive observations and corrections. Faster sampling gives better feedback performance because the actuator is observed and corrected on a shorter time scale. Good performances are usually obtained for sampling rates that are 20 to 30 times higher than the closed loop bandwidth. The LHC orbit acquisition has been designed to sample at 10 Hz, it is therefore already clear that the closed loop bandwidth of the system will be limited to  $\sim 1$  Hz.

#### 5.2 Discrete Controller Design

The model of our system uses discrete time transfer functions with a sampling time of 100 ms. The time constants of power converter and magnet are set to 100 ms (assuming operation around injection energy), while the time constants of the vacuum chamber is set to 10 ms. The total system delay is set to one sampling interval, which adds an extra pole at the origin to the transfer function. The discrete overall transfer function is

$$G(z) = \frac{0.59z + 0.04}{z^2(z - 0.36)} \tag{4}$$

A PI controller can be designed for this transfer function using the standard loop shaping technique of "pole-zero" matching. The PI controller transfer function which is given by

$$H(z) = K_p + \frac{K_i}{z - 1} = K_p \frac{z - (1 - K_i/K_p)}{z - 1}$$
(5)

introduces a pole at z = 1 and leaves a zero to choose. For  $K_i = 0.632K_p$ , the pole of the plant is cancelled with the zero of the controller, leaving the gain  $K_p$  free. For large  $K_p$  one obtains good disturbance rejection (higher gain),



Figure 2: Root locus plot for the LHC orbit feedback loop using PI control with  $K_p$  varied between 0 and infinity and  $K_i = 0.632K_p$ . Poles are indicated by (x) and zeros by a (0).



Figure 3: Bode plots corresponding Fig. 2 for  $K_p = 1$ .

but there is an increased risk of making the system unstable. A standard technique to study the stability of the system is the Root-locus plot, shown for our system in Fig. 2 when the gain  $K_p$  is varied from zero to infinity.

For  $K_p$  set to 1, the system gain is ~ 10 at 0.1 Hz, but there is no reduction for signals at 1 Hz. This can be deduced from the corresponding Bode plot in Fig. 3. This figure also indicates a phase margin of 67 degrees and the gain margin of 3.1. The gain margin of a closed loop system is the factor by which the gain can be increased before the system becomes instable. The phase margin is the difference between the induced phase shift and 180 degrees when the gain of the system is equal to 1.

Other controllers and optimisation procedures are listed in Table 1. For example, instead of fixing the zero of the controller to coincide with the pole of the plant, it can be considered as an additional variable to improve the gain at 1 Hz. The optimised PI controller yields better gain at 1 Hz at the cost of a reduced gain at 0.1 Hz. Alternatively a PID controller can be designed to increase the bandwidth, but the improvement (30% higher gain at 1 Hz) is only marginal. It should also be noted that a derivative term makes the system more sensitive to BPM noise.

The time delay can be compensated with a Smith predic-

PI Controller	Gain	Gain	Sampling
	at 1 Hz	at 0.1 Hz	rate (Hz)
$K_i = 0.632 K_p$	1.0	10.6	10
Optimised	1.2	8.0	10
Smith predictor	2.2	22.2	10
Optimised	1.4	10.5	20
Smith predictor	6.2	62.6	20

Table 1: Feedback gains at 1 and 0.1 Hz for different controller designs.

tor that is feeding back a simulated plant output to cancel the true plant output and then adding in a simulated plant output without the delay. A Smith predictor with PI controller yields considerably higher gains, but is more difficult to tune than a simple PI controller.

From Table 1 it is also evident that a sampling frequency of 20 Hz gives a consistently better performance in all cases.

#### 6 CONCLUSIONS

It is possible to design a site wide orbit feedback loop for the LHC that can eliminate orbit distortions for both beams during injection and at the beginning of the ramp. A simple PI controller is sufficient to place the time constants of the system at their desired locations and provide sufficient error reduction at frequencies in the range of 0.1 Hz. A considerably better performance can be obtained when the sampling frequency is increased and/or when the total time delay is reduced.

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