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Orbit Corrector Magnets and Beam Energy

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Abstract

The influence of orbit corrector magnets on the beam energy has been evaluated for LEP using simple models as well as simulations with the MAD program. The results of the simulation could be confirmed by two dedicated experiments where energy shifts due to changes in horizontal corrector patterns of 1 to 2 MeV were measured by resonant depolarization.

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1 Introduction

At LEP the beam energy is measured routinely with a relative accuracy of 10^{-5} using resonant depolarization [1]. Despite many efforts, no model of the LEP energy is able to describe in detail all the data that has been accumulated so far [2, 3]. This is most likely due to the fact that the model is incomplete and that the influence of certain parameters is not known well enough. The effects of corrector settings were expected to be small $\mathcal{O}(1\ \text{MeV})$ and were not included in the 1993 beam energy model [2]. A systematic error was however estimated for their effects. This procedure was no longer adequate for the 1995 LEP energy scan where systematic differences of horizontal corrector settings were found between physics fills and calibration runs as well as between crucial calibrations at the beginning and end of fills. To control the impact of such differences on the beam energy uncertainties, a more detailed understanding of the corrector effects was required.

This note describes the theoretical and experimental studies that have been made to understand the relation between beam energy and corrector settings.

2 Orbit Distortions and Beam Energy

To begin this study two simple models to predict the effects of dipole kicks on the beam energy will be described. The first model explains energy shifts with a change of the orbit length while the second assumes that the kicks modify the effective bending field of the ring

2.1 Orbit Lengthening Model

When the ideal orbit is distorted by a single dipole kick θ, the closed orbit distortion u is a function of the path length s along the ideal orbit given by :

$$u(s) = \theta\,\frac{\sqrt{\beta_u(s_0)\beta_u(s)}}{2\sin(\pi Q_u)}\cos(|\phi_u(s)-\phi_u(s_0)|-\pi Q_u) \tag{1}$$

$\beta_u(s)$ and $\phi_u(s)$ are the betatron function and the betatron phase. Q_u is the machine tune. s_0 is the longitudinal position of the kick θ. This expression is valid in both transverse planes ($u = x, y$). Such an orbit distortion modifies the orbit length and possibly the average beam energy. It is important to note that a single horizontal kick θ in does not add to the bending of the dipole magnets since the kick is compensated by the lattice on the closed orbit.

In a section where the curvature ρ is locally constant, the beam position can be expressed in polar coordinates $r = \rho + u$ and ϕ. While the path length on the ideal orbit is $ds = \rho d\phi$, the path length dl on the distorted orbit becomes (Figure 1) :

$$dl = \sqrt{r^2 + \left(\frac{dr}{d\phi}\right)^2}\,d\phi \tag{2}$$

The change of the path length dL due to the distortion

$$dL = dl - ds = \sqrt{r^2 + \left(\frac{dr}{d\phi}\right)^2}\,d\phi - ds \tag{3}$$

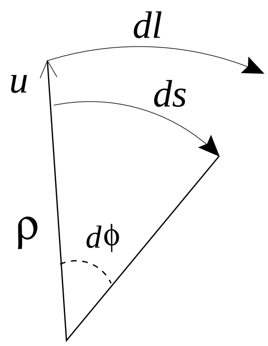

Figure 1: Path lengths on the ideal (ds) and on the distorted (dl) orbit.

can be expanded to yield

$$dL \approx \left(\frac{u}{\rho} + \frac{1}{2} \left(\frac{du}{ds} \right)^2 \right) ds \tag{4}$$

where only the leading terms have been kept. The total path length change Λ due to the closed orbit distortion is obtained from an integral over the whole ring circumference,

$$\Lambda = \oint dL = \Lambda_1 + \Lambda_2 \tag{5}$$

where we have split Λ into a linear lengthening Λ_1 and a quadratic lengthening Λ_2 [4] :

$$\Lambda_1 = \oint \frac{u}{\rho}\, ds \qquad\qquad \Lambda_2 = \frac{1}{2} \oint \left(\frac{du}{ds} \right)^2 ds \tag{6}$$

Λ_1 is due to the shift of the beam position. Straight sections do not contribute to Λ_1 since $1/\rho = 0$. Λ_2 represents the length change due to the closed orbit RMS.

When Λ is evaluated for the closed orbit distortion given by Equation 1, the linear lengthening reduces to a simple expression depending only on the dispersion D_u and the kick θ [4] :

$$\Lambda_1 = D_u(s_0)\, \theta \tag{7}$$

while the quadratic orbit lengthening can be approximated by :

$$\Lambda_2 \approx \frac{8 Q_u^2 \hat{u}^2}{L_0} \tag{8}$$

$\hat{u}$ stands for the maximum amplitude of the closed orbit excursion. L_0 is the length of the ideal orbit.

Since the RF frequency constrains the length of the orbit to remain constant, the beam must change its average radial position to accommodate the lengthening Λ. This in turn leads to a change of the average beam energy E :

$$\frac{\Delta E}{E} = -\frac{1}{\alpha} \frac{\Lambda_1 + \Lambda_2}{L_0} \tag{9}$$

where α is the momentum compaction factor. This relation is only correct for a linear machine (no sextupoles). In a real machine where the sextupoles are used to compensate the momentum dependence of the tune, ΔE receives an additional contribution from the sextupole deflections θ_s leading finally to [4] :

$$\frac{\Delta E}{E} = -\frac{1}{\alpha} \frac{\Lambda_1 + \Lambda_2}{L_0} - \frac{\sum \theta_s}{2\pi} \tag{10}$$

2.2 Bending Field Model

Alternatively the fields corresponding to (some) horizontal kicks may contribute to the total bending field. The energy change due to a kick θ would then be given by :

$$\frac{\Delta E}{E} = -\frac{\theta}{2\pi} \tag{11}$$

This assumption is of course slightly naive. For correctors, at least a certain fraction of the horizontal kicks compensate the orbit distortions introduced by misaligned quadrupoles. Their fields are therefore not contributing to the bending field. This model may however apply to situations where the corrector pattern is very asymmetric, i.e. when many corrector kicks are of the same sign and evenly distributed over the whole ring or over one octant. A certain fraction of the corrector fields may then contribute to the bending.

The two models described here will be later referred to as the "orbit lengthening" and the "bending" model. Two useful quantities can be defined for horizontal correctors, the linear orbit lengthening Λ_{1C} and the total deflection Θ_C :

$$\Lambda_{1C} = \sum_i D_x^i \theta_{cx}^i \qquad\qquad \Theta_C = \sum_i \theta_{cx}^i \tag{12}$$

D_x^i is the horizontal dispersion and θ_{cx}^i the kick at the ith corrector. The sums run over all horizontal correctors in the machine.

It is interesting to consider for LEP the case of two horizontal corrector settings a and b which give about the same closed orbit RMS and which differ only in the regular arc cells. If the effect of the sextupoles is assumed to be roughly identical for both settings, the energy difference predicted by the orbit lengthening model is :

$$\frac{\Delta E}{E} = -\frac{1}{\alpha L_0}\left(\Lambda_{1C}^a - \Lambda_{1C}^b\right) = -\frac{D_x^m}{\alpha L_0}\left(\Theta_C^a - \Theta_C^b\right) \simeq -\frac{D_x^m}{< D_x >}\left(\frac{\Theta_C^a - \Theta_C^b}{2\pi}\right) \tag{13}$$

D_x^m is the dispersion at the arc correctors and $< D_x >$ is the average dispersion. Equation 13 shows that the energy shifts predicted by the two models differ only by a factor $D_x^m/ < D_x > \simeq 1.3$ in this situation.

3 Simulations of Corrector Effects

A simulation of LEP with the MAD program [5] was carried out to test the influence of the correctors and the validity of the models presented in the previous section. All simulations have been made for a positron beam at a nominal energy E_0 of 20 GeV using the 1995 LEP bunch train optics (l05p46v6). The energy was chosen to avoid large interferences between the orbit correction procedure and the horizontal energy sawtooth. The vertical orbit was kept flat (no bunch train separation bumps). The average energy was obtained from an integral over the whole ring of the local energy deviation $\delta E(s)$:

$$\Delta E = \frac{\oint \delta E(s)\, ds}{L_0} \tag{14}$$

The average beam energy, $E_0 + \Delta E$, should correspond to the energy measured by resonant depolarization.

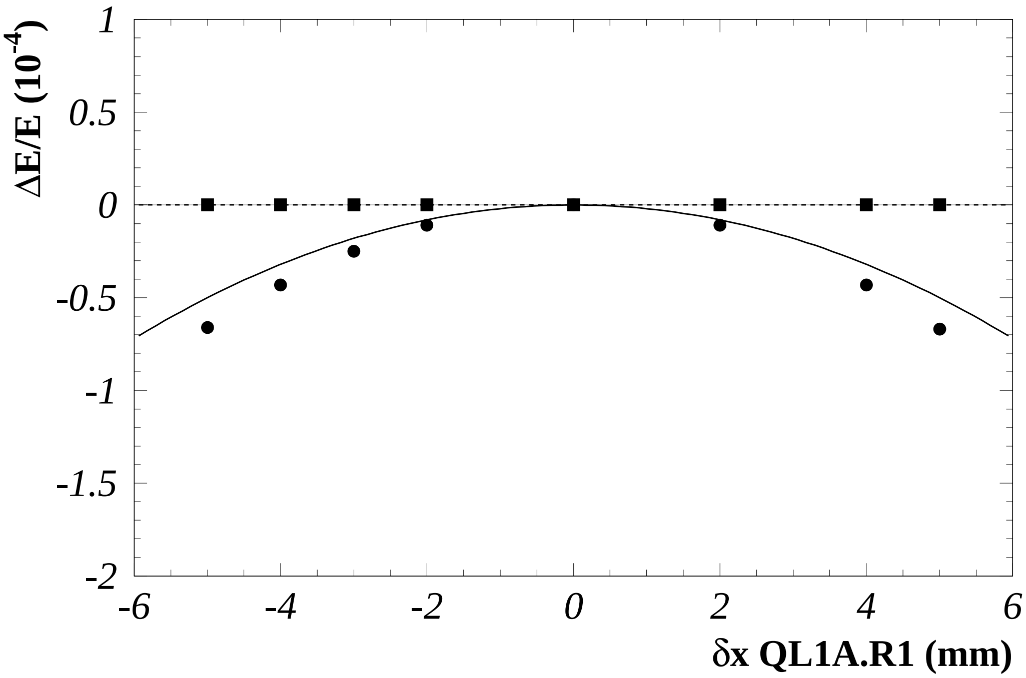

Figure 2: Relative energy change $\Delta E/E$ due to a horizontal misalignment of the straight section quadrupole QL1A.R1 when the closed orbit is left uncorrected. The simulation was performed with the sextupoles on (square dots) and off (round dots). The solid (dashed) line is the prediction for $\Delta E/E$ due to the lengthening Λ (Λ_1) using Equations 15 and 16 with the following parameters for QL1A.R1 : kl = -0.042 m^{-1} , $\beta_x(s_0) = 47$ m , $\beta_x^{max} = 122$ m , $D_x(s_0) = 0$ m and $Q_x = 90.29$.

3.1 Simulation of Single Deflections

In a first step, the energy shift was evaluated for a perfect machine with a single misaligned quadrupole. The orbit was left uncorrected. For a linear machine the orbit lengthenings Λ_1 and Λ_2 can be evaluated analytically from Equations 7 and 8. For a quadrupole of strength k and length l which is displaced by δu, the kick is $\theta = kl\,\delta u$ and the lengthenings are :

$$\Lambda_1 = D_u(s_0)\,kl\,\delta u \tag{15}$$

and :

$$\Lambda_2 \approx 2\left[\frac{Q_u\,kl\,\delta u}{\sin(\pi Q_u)}\right]^2 \frac{\beta_u(s_0)\beta_u^{max}}{L_0} \tag{16}$$

β_u^{max} is the maximum value of the betatron function in the arcs. The results of the MAD simulations are compared with the orbit lengthening model in Figures 2 and 3. When the sextupoles are off, the qualitative and quantitative agreement of the simulations with the simple analytical estimate is quite good. The simulation clearly disagrees with the bending model where a linear dependence of the energy on the misalignment would be expected. When the sextupoles are on, the energy shift due to Λ_2 is almost perfectly compensated by the sextupoles and the energy change is given by :

$$\frac{\Delta E}{E} \simeq -\frac{\Lambda_1}{\alpha L_0} \tag{17}$$

As a consequence, kicks located in areas without dispersion do not affect the energy significantly (see Figure 2). The influence of the vertical plane is very small. The results of the simulation

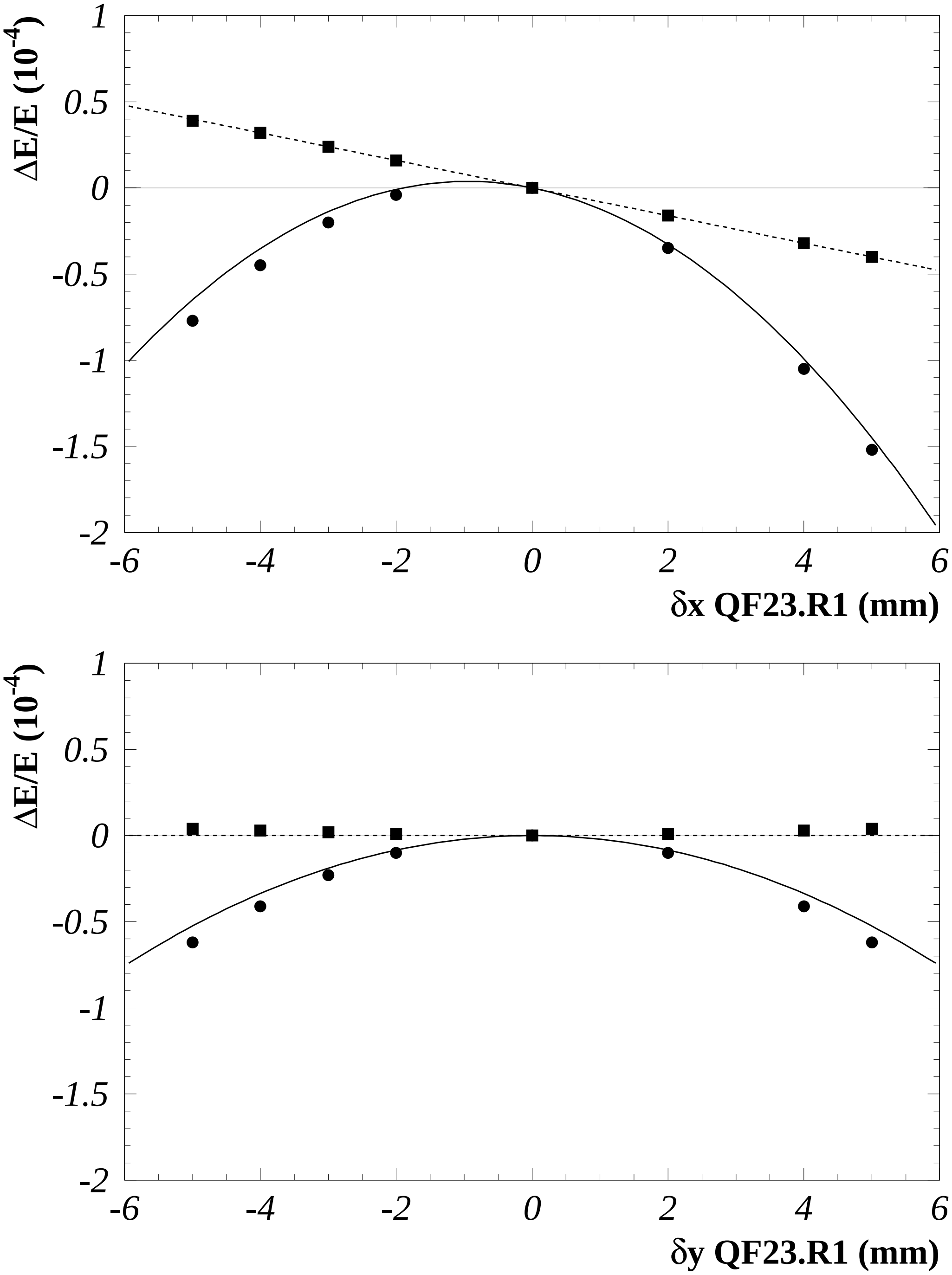

Figure 3: Relative energy change $\Delta E/E$ due to a horizontal (top) and vertical (bottom) misalignment of the arc quadrupole QF23.R1 when the closed orbit is left uncorrected. The simulation was performed with sextupoles on (square dots) and off (round dots). The solid (dashed) line is the prediction for $\Delta E/E$ due to the lengthening Λ (Λ_1) using Equations 15 and 16 with the following parameters for QF23.R1 : $kl = 0.0347$ m^{-1}, $\beta_x(s_0) = \beta_x^{max} = 122$ m, $D_x(s_0) = 1.14$ m, $Q_x = 90.29$, $\beta_y(s_0) = 41$ m, $\beta_y^{max} = 153$ m, $D_y(s_0) = 0$ m and $Q_y = 76.19$.

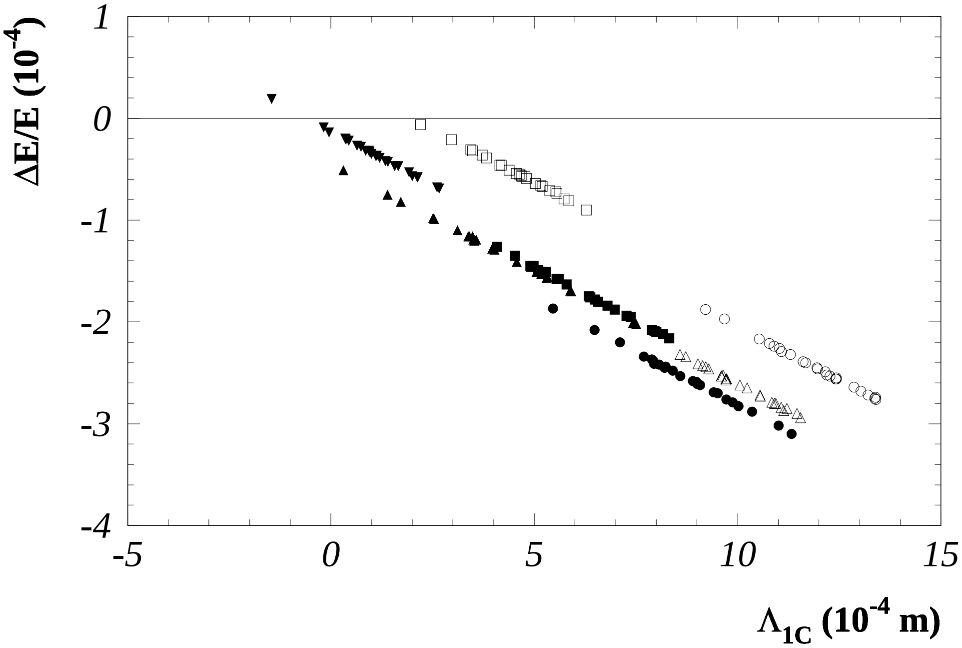

Figure 4: Relative energy shifts $\Delta E/E$ with respect to the nominal energy as a function of the orbit lengthening Λ_{1C} from horizontal corrector kicks. Each symbol type corresponds to the same simulated machine. The different points shown for the each symbol correspond to different corrector patterns.

apply of course to deflections of any origin (correctors, quadrupoles ...). A similar influence of the sextupoles has also been observed for the energy shifts due to the horizontal Pretzel orbits [6].

3.2 Simulation of a Complete Machine

In a second step, the simulation of a realistic machine was made with alignment and field errors applied to all elements. The most important imperfections for this study are the misalignments of the quadrupoles and the BPMs. The vertical and horizontal misalignments of the quadrupoles were set to 0.15 mm RMS and 0.3 mm RMS. The BPMs were misaligned with respect to the quadrupole axis by 0.2 mm RMS. The relative RMS field error of the main dipoles was set to $7 \cdot 10^{-4}$. Field errors of $\sim 10^{-4}$ where used for all other magnetic elements.

The influence of the corrector pattern used by the orbit corrections was tested on 7 different machines. For each machine the orbit was corrected to a target RMS of 0.5 mm in the horizontal and 0.4 mm in the vertical plane. The tolerance on the RMS was about 10%. The orbit correction was repeated 20 times for each machine with a different set of corrector magnets. To force the correction algorithm to modify the corrector pattern, a different set of 10% of the BPMs was disabled randomly for each correction. Figure 4 shows that the relative energy shift $\Delta E/E$ correlates very well with the linear lengthening Λ_{1C} (Equation 13) :

$$\frac{\Delta E}{E} \sim \kappa\, \Lambda_{1C} \tag{18}$$

This reproduces the results obtained for a single kick. The average slope for the 7 machines shown in Figure 4 is $\kappa = -0.208 \pm 0.003\,(m^{-1})$. This value is very close to $\kappa = -1/(\alpha L_0) = -0.202\,(m^{-1})$ suggested by the orbit lengthening model when only Λ_1 is contributing to $\Delta E/E$.

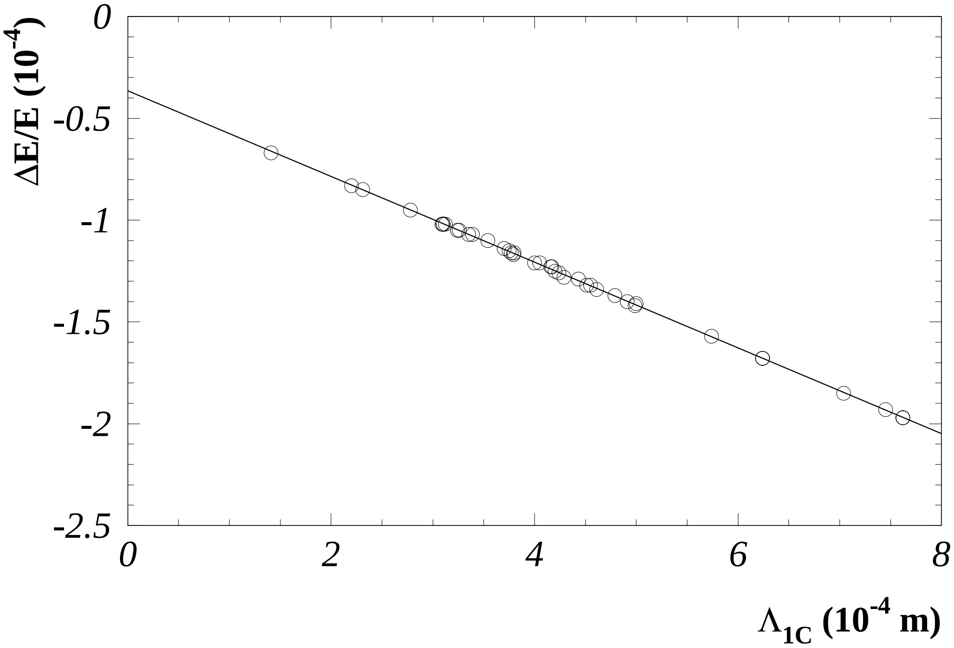

Figure 5: Relative energy shifts $\Delta E/E$ as a function of the linear orbit lengthening Λ_{1C} from horizontal corrector kicks. All points correspond to the same simulated machine. The RMS of the orbit correction was varied randomly between 0.4 and 0.7 mm for the horizontal, between 0.3 and 0.6 mm for the vertical plane. Each point in the figure corresponds therefore to a different corrector pattern and to a different closed orbit RMS. The fitted line has a slope of -0.210 (m^{-1}).

The correlation of $\Delta E/E$ with the prediction of the bending model is much poorer. In addition, the slope is not correct, as expected if the orbit lengthening is the cause of the energy shifts.

Since in reality orbits are not always corrected to the same RMS values, the simulation was repeated for one machine to evaluate the influence of the orbit RMS on the beam energy. 40 orbit corrections were applied with target RMS values ranging between 0.4 and 0.7 mm in the horizontal and 0.3 to 0.6 mm in the vertical plane. Figure 5 shows that $\Delta E/E$ remains proportional to Λ_{1C}. The fluctuations of the orbit RMS do not perturb the good correlation : this is probably due to the influence of the sextupoles.

In figures 4 and 5 Λ_{1C} is almost always *positive*. The reason for this preference is not understood, but it is also observed with the LEP corrector settings between 1993 and 1996.

3.3 Simulation Summary

The MAD simulations clearly indicate that a modification of the horizontal corrector settings leads to an energy shift $\Delta E_{\Lambda 1}$ proportional to the linear orbit lengthening :

$$\frac{\Delta E_{\Lambda 1}}{E} = -\frac{1}{\alpha L_0} \sum_i D_x^i \, \Delta\theta_{cx}^i \tag{19}$$

where $\Delta\theta_{cx}^i$ is the change of the kick of the ith horizontal orbit corrector.

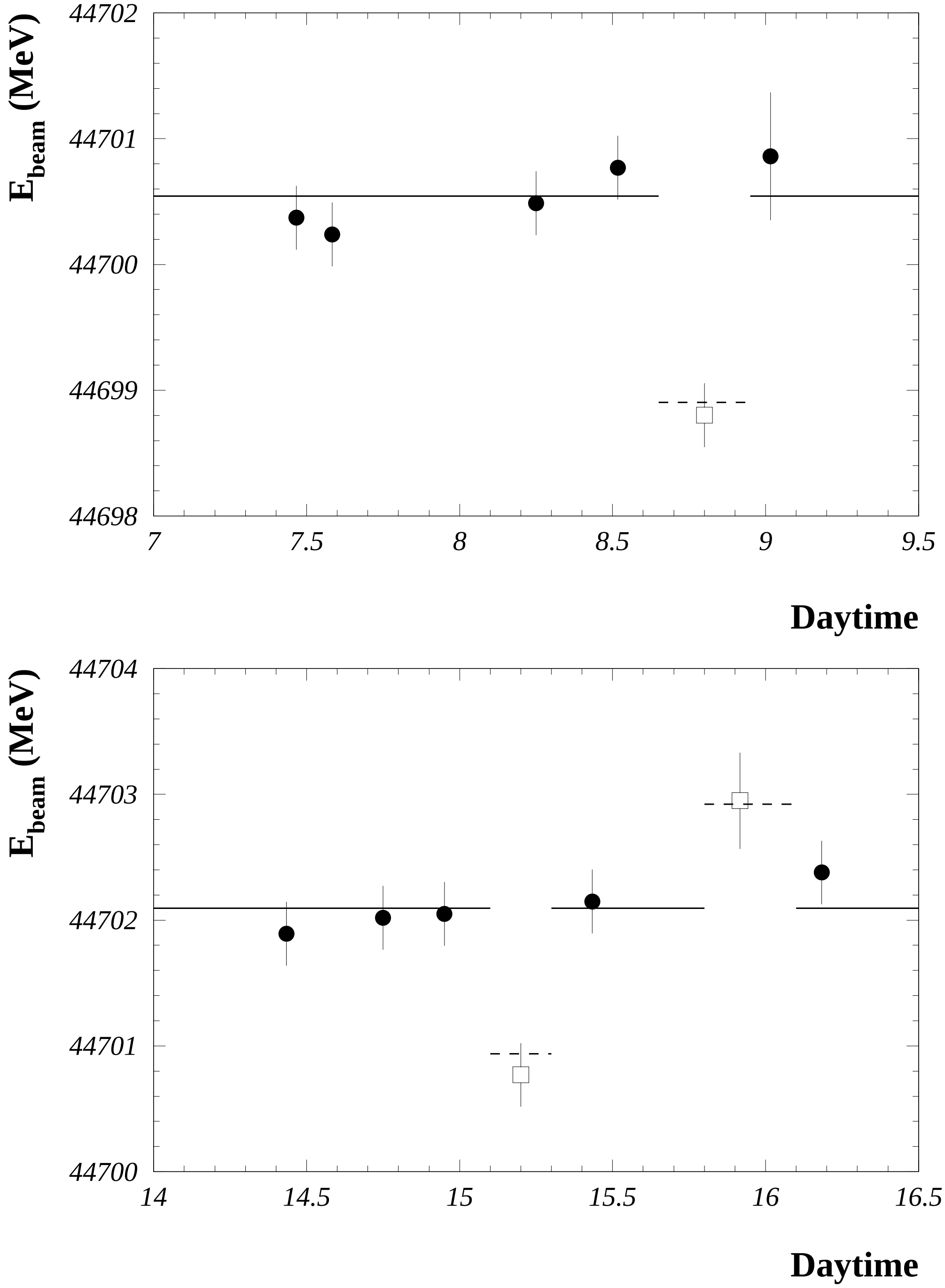

Figure 6: Evolution of the beam energy E_{beam} as a function of time in LEP fills 3702 (top) and 3719 (bottom). E_{beam} is corrected for tides and magnet temperature. All measurements plotted as circles correspond to the reference orbit with a fixed corrector pattern. The solid lines indicates the average energy on the reference orbit. Measurements performed with different corrector settings are indicated by squares. The dashed lines represent the energy shifts predicted from the orbit lengthening due to the change of corrector patterns with respect to the reference orbit.

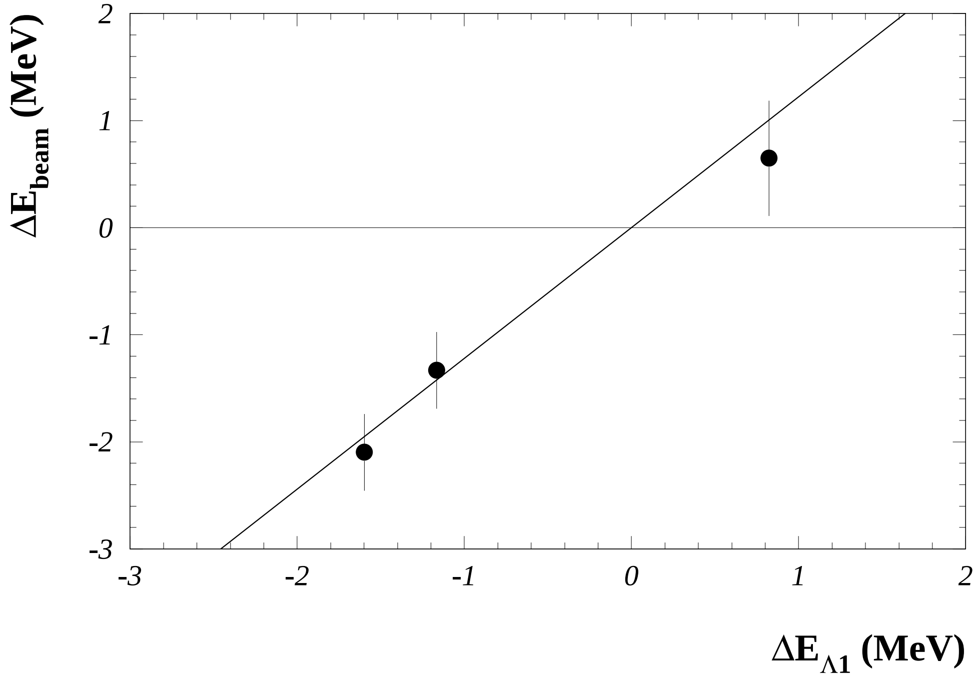

Figure 7: Correlation of the measured (ΔE_{beam}) and the predicted ($\Delta E_{\Lambda 1}$) energy change due to horizontal correctors. The slope is 1.22 ± 0.18.

4 Experiments

The influence of the horizontal correctors was tested experimentally in LEP fills 3702 and 3719. In both cases the beam energy of 44.7 GeV ("Peak-2") was first determined accurately by resonant depolarization for a "reference orbit" corresponding to the initial horizontal corrector pattern. The corrector strengths were then modified to obtain a new pattern for which a measurable energy change was predicted from the orbit lengthening. The beam energy was measured for this new corrector pattern. To avoid biases, the settings of the reference orbit were then restored and the energy remeasured on the reference orbit. This procedure was repeated twice in fill 3719. Figure 6 shows the evolution of the beam energy as a function of time for the two experiments. The energy shifts due to the change of horizontal corrector settings are clearly visible. $\Delta E_{\Lambda 1}$, the expected energy change with respect to the reference orbit, is correlated with the measured energy change in figure 7. A fit gives a slope of 1.22 ± 0.18 which confirms the validity of the model.

If the energy shift is analysed with the bending model, a good correlation is also obtained. A fit of the measured versus the predicted energy change gives a slope of 1.33 ± 0.18. Although the agreement is slightly worse than for the orbit lengthening model, the experiments cannot clearly discriminate between the two models.

5 Discussion

The MAD simulations indicate that the orbit lengthening model describes best the energy shifts. Yet it cannot be excluded that the more complicated and subtle orbit corrections procedures used in reality for LEP give some weight to the bending model. In 1993 for example, it has been observed that in two octants with weaker bending fields the correctors seemed to add to

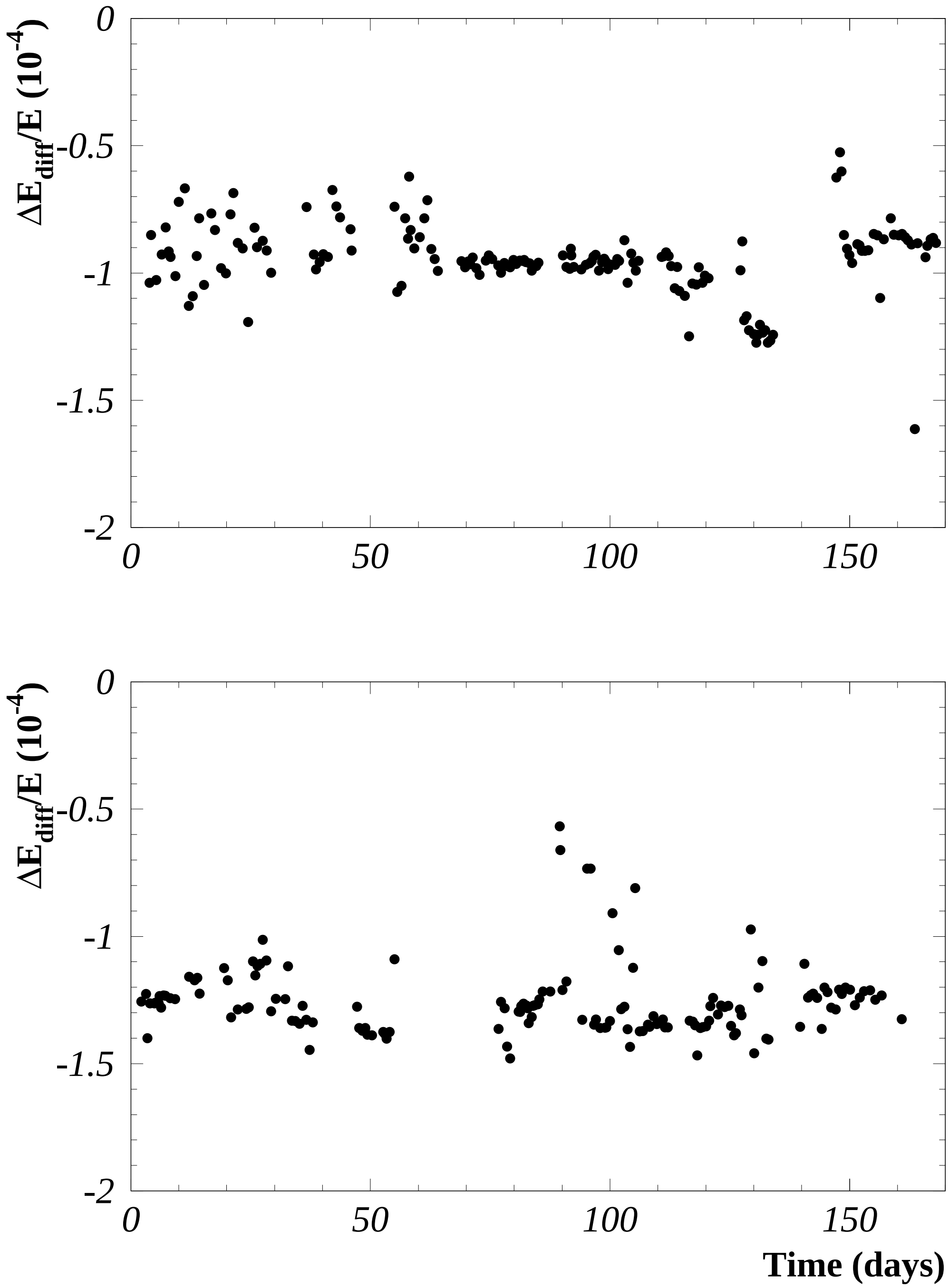

Figure 8: The difference of the energy shifts $\Delta E_{diff}/E$ due to horizontal correctors predicted by the bending and orbit lengthening models are shown as a function of time in 1994 (top) and 1995 (bottom). Each point represents the average over one fill. The time scales start on May 1^{rst} for both figures.

Table 1: Relative energy changes due to horizontal correctors between the begin and end of fill calibration. For a positive shift the correctors contribute more to the energy at the end of the fill. ΔE_Θ and ΔE_Θ^{ARC} are the predictions from the bending model when all correctors, respectively only ARC correctors, are taken into account.

Fill	$\Delta E_{\Lambda 1}/E$ (10^{-5})	$\Delta E_\Theta/E$ (10^{-5})	$\Delta E_\Theta^{ARC}/E$ (10^{-5})
3022	0.5	0.8	0.5
3029	6.8	5.9	5.2
3030	5.7	4.7	4.2
3036	-0.9	-1.0	-0.8

the bending of the dipoles [7].

From the logged corrector settings, the energy shifts predicted by both models can be reconstructed for the LEP energy scans. The differences between the two models can be used as estimates for systematic errors. Figure 8 shows the difference of the predictions (ΔE_{diff}) for the 1994 and 1995 LEP runs. The RMS scatters of $\Delta E_{diff}/E$ are small : $1.9 \cdot 10^{-5}$ for 1994 and $1.4 \cdot 10^{-5}$ for 1995. The average value of ΔE_{diff} does not affect the LEP energy error since the absolute energy scale is set by resonant depolarization.

In 1995 four fills have been calibrated at the beginning and at the end of the fill, with different corrector settings for the two calibrations. Table 1 gives the predicted contribution of the horizontal correctors to the beam energy difference between the begin and end of fill calibration. For two fills the contribution reaches 2.5 MeV.

If the horizontal correctors change the length of the orbit, they will also influence the central RF frequency f_{RFC} as well as the beam positions X_{ARC} obtained from the BOM system [7, 8]. Both f_{RFC} and X_{ARC} depend on Λ, but as long as the orbit RMS does not vary too much, Λ_2 remains roughly constant and all changes should correlate to Λ_{1C}. A model that includes either f_{RFC} or X_{ARC} should then automatically take into account some of the effects of horizontal correctors. The existing data is not accurate enough to observe such correlations between Λ_{1C} and X_{ARC}.

Finally the continuous slow movement of the quadrupoles in LEP limits the accuracy of a long term correction of horizontal corrector effects since the position change of the quadrupoles should also be taken into account.

6 Conclusion

Simulations with MAD indicate that orbit corrections can modify the LEP beam energy. The energy change is proportional to the linear orbit lengthening from horizontal correctors, a quantity that can be easily evaluated for a given corrector setting. An alternative model where the energy change would be due to a modification of the bending field is not favoured by the MAD simulations.

Two controlled experiments have shown that horizontal correctors influence the beam energy. The data is compatible with both models that have been presented. For this reason, the impact on the beam energy must be calculated for both corrector models. Differences should be used as systematic errors on the beam energy. Fortunately the predictions from the two models do not differ very much and such systematic errors will not be very large.

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