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Influence of Dispersion and Collision Offsets on the Centre-of-Mass Energy at LEP

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Abstract

The effect of dispersion and collision offsets at the interaction points of LEP on the centre-of-mass energy has been calculated for various collision and dispersion patterns. Different dispersions of the two beams can lead to variation of the centre-of-mass energy spread of about 5% in the case of bunch train operation. The centre-of-mass energy is systematically shifted if the beams collide with some offsets and if the dispersions of the two beams are not identical. The centre-of-mass energy can differ by almost 50 MeV at one interaction point among the 4 collisions of trains of 4 bunches. Trains can be collided such that energy shifts within a train cancel each other. However, small systematic offsets may lead to systematic errors that are difficult to control at the level required to achieve the goals of the 1995 energy scan.

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1 Introduction

In 1995 it is foreseen to operate LEP with short bunch trains [1]. Vertical electrostatic closed orbit bumps are used in this scheme to separate the beams at the parasitic collision points. As a consequence vertical dispersion is produced at the interaction points (IPs). The vertical dispersion of the two beams is roughly equal but of opposite sign [2]. This affects the energy spread in the centre-of-mass system (CM) as well as the average CM energy at each IP. Because of the parasitic encounters, the train shape is "banana-like" [3] in the vertical plane and the bunches collide with transverse collision offset. For these reasons, we have evaluated the impact of unequal dispersion and collision offsets at the IPs on the CM energy distribution.

2 Centre-of-Mass Energy Corrections without Collision Offsets

The parameters of the beams at the IP can be calculated using normalized particle density distributions. We will use Gaussian distributions to represent the particle densities in phase space [5]. The density distribution at the IP is a function of the relative beam energy deviations $\epsilon = (E - E_0)/E_0$ of the two beams (labeled 1 and 2) and of the beam interaction position u. For simplicity we will work with only one space dimension u which may be horizontal (x) or vertical (y). The particle density is expressed by

$$P(u, \epsilon_1, \epsilon_2) = \frac{1}{N} g(u, \epsilon_1) g(u, \epsilon_2) \tag{1}$$

$$g(u, \epsilon_i) = \exp\left[-\frac{1}{2}\left(\frac{\epsilon_i^2}{\sigma_\epsilon^2} + \left(\frac{u - D_{ui}\epsilon_i}{\sigma_u}\right)^2\right)\right] \tag{2}$$

σ_ϵ is the relative beam energy spread, σ_u is the betatron component of the beam size which depends on the betatron function at the IP (β_u^*) and the beam emittance (ε_u) :

$$\sigma_u = \sqrt{\beta_u^* \varepsilon_u} \tag{3}$$

The two beams are labeled by (i). We assume that only the dispersion D_{ui} differs for the two beams. The total spot size σ_{Bi} of beam (i) at the IP is

$$\sigma_{Bi}^2 = \sigma_u^2 + (D_{ui}\sigma_\epsilon)^2 \tag{4}$$

The normalization factor N is given by

$$N = (2\pi)^{3/2} \frac{\sigma_\epsilon^2 \sigma_u^2}{\sigma_{B1}^2 + \sigma_{B2}^2} \tag{5}$$

to ensure that

$$\frac{1}{N} \int\int\int g(u, \epsilon_1) g(u, \epsilon_2) d\epsilon_1 d\epsilon_2 du = 1 \tag{6}$$

With this a normalization $P(u, \epsilon_1, \epsilon_2)$ represents a luminosity weighted probability density. Any physical quantity that is integrated over phase space with this normalization will be automatically weighted by luminosity.

The luminosity weighted CM energy shift ΔE_{CM} and spread $\sigma_{E_{CM}}$ are obtained by integration of the energy of a collision $(\epsilon_1 + \epsilon_2)$ over the collision area and over all energies :

$$\Delta E_{CM} = \frac{E_0}{N} \int\int\int (\epsilon_1 + \epsilon_2) g(u, \epsilon_1) g(u, \epsilon_2) d\epsilon_1 d\epsilon_2 du \tag{7}$$

$$\sigma^2_{E_{CM}} = \frac{E_0^2}{N}\int\int\int(\epsilon_1+\epsilon_2)^2 g(u,\epsilon_1)g(u,\epsilon_2)d\epsilon_1 d\epsilon_2 du - \Delta E^2_{CM} \tag{8}$$

The exponent of equation 2 can be rewritten in a more convenient form for the integrations :

$$\frac{\epsilon_i^2}{\sigma_\epsilon^2} + \left(\frac{u - D_{ui}\epsilon_i}{\sigma_u}\right)^2 = \frac{u^2}{\sigma^2_{Bi}} + \left(\frac{\sigma_{Bi}\sigma_\epsilon}{\sigma_u}\right)^2\left[\frac{\epsilon_i}{\sigma^2_\epsilon} - \frac{uD_{ui}}{\sigma^2_{Bi}}\right]^2 \tag{9}$$

The evaluation of equations 7 and 8 involves integrals that can be found in standard tables.

The final results for head on collisions are ($\sigma_E = \sigma_\epsilon E_0$) :

$$\Delta E_{CM} = 0 \tag{10}$$

$$\sigma^2_{E_{CM}} = \sigma^2_E\left[\frac{\sigma^2_\epsilon(D_{u1}+D_{u2})^2 + 4\sigma^2_u}{\sigma^2_{B1}+\sigma^2_{B2}}\right] \tag{11}$$

The CM energy shift ΔE_{CM} **vanishes** for any value of the dispersion. We can simplify these equations if the dispersions have the same **absolute** value. In that case we have

$$\sigma_B \equiv \sigma_{B1} = \sigma_{B2} \tag{12}$$

which leads to

$$\sigma^2_{E_{CM}} = \frac{\sigma^2_E}{2\sigma^2_B}\left[\sigma^2_\epsilon\left(D_{u1}+D_{u2}\right)^2 + 4\sigma^2_u\right] \tag{13}$$

The two corresponding possibilities are now easily evaluated.

- **A** : the dispersion is **identical** for both beams ($D_{u1} = D_{u2}$). In that case the energy spreads of the two beams add up incoherently :

  $$\sigma^2_{E_{CM}} = 2\sigma^2_E \tag{14}$$

  This surprising result [5, 6] is due to the fact that while there is some correlation between the particle energies of both beams, the increase in total beam spot size cancels this effect again. However there is a correlation between the transverse collision point position and CM energy.

- **B** : the dispersions have the **opposite sign** ($D_{u1} = D_u = -D_{u2}$). We now get a correction which depends on the ratio σ_u/σ_B (see also [5]) :

  $$\sigma^2_{E_{CM}} = 2\sigma^2_E\left[1 + \left(\frac{D_u\sigma_\epsilon}{\sigma_u}\right)^2\right]^{-1} = 2\sigma^2_E\,\frac{\sigma^2_u}{\sigma^2_B} \tag{15}$$

  In the particular case where σ_u vanishes, one obtains a monochromatic CM energy because of the total anti-correlation between the energies of the colliding particles.

  We can make some numerical estimates for the order of magnitudes that can be expected with trains of 4 bunches. For the vertical plane with $\varepsilon_y = 1$ nm, $\beta^*_y = 5$ cm, $D_y = 2$ mm [2] and an energy spread of $\sigma_\epsilon \approx 10^{-3}$ (when the emittance wigglers are used), one obtains a correction of $\sigma_u/\sigma_B = 0.96$ with respect to the situation where the energy spreads add up incoherently. This effect alone shifts the width of the Z by about 0.4 MeV. This correction varies in time since the beam sizes and the energy spread evolve during the fill.

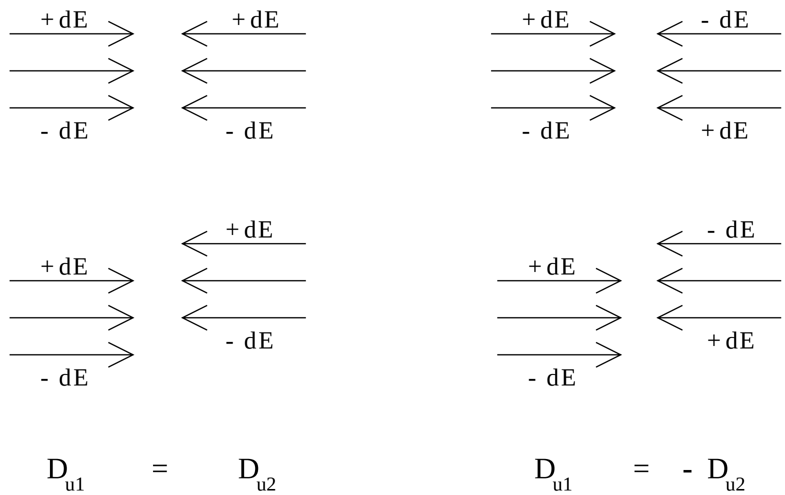

Figure 1: Schematic view of the collision of two beams with vanishing transverse beam size but non-zero dispersion. The collision points for 3 particles with energy deviations +dE, 0 and -dE are shown for various situations. On the left side, the case of equal dispersion is considered. When the beams collide with offsets, the particles which miss the collisions come from the high energy tail of one beam and from the low energy tail of the other beam. There is no energy shift. In the situation of opposite dispersion (right), we obtain a monochromatic beam in the CM. When there is a collision offset the low or high energy particles (depending on the sign of this offset) of both beams are not colliding, which results in an energy shift in the CM.

3 Centre-of-Mass Energy Corrections for Collisions with Offsets

For the case where the beams collide with an offset at the IP, the CM energy spread and shift are recalculated by replacing u by $u - u_0$ in $g(u, \epsilon_1)$ and by $u + u_0$ in $g(u, \epsilon_2)$ and re-evaluating the normalization factor N. This corresponds to a total separation of the beams at the IP of $2u_0$. The final expression for the CM energy shift and spread are

$$\Delta E_{CM} = -2u_0 \frac{\sigma_E^2 (D_{u1} - D_{u2})}{E_0(\sigma_{B1}^2 + \sigma_{B2}^2)} \tag{16}$$

$$\sigma_{E_{CM}}^2 = \sigma_E^2 \left[\frac{\sigma_\epsilon^2 (D_{u1} + D_{u2})^2 + 4\sigma_u^2}{\sigma_{B1}^2 + \sigma_{B2}^2} \right] \tag{17}$$

The spread in CM energy is not affected by the collision offset. The results obtained previously for $u_0 = 0$ are still valid.

A CM energy shift is produced if the dispersion of the beams is different **and** if the beams collide with an offset. This effect can be explained with the simple picture shown in figure 1. If the dispersion is identical for both beams and there are collision offsets, we lose particles from the high and the low energy tail. If the dispersion has the opposite sign, we lose the high (or low) energy particles only, depending on the signs of the dispersion and the separation. This will obviously produce a shift of the mean CM energy.

We can again study the two simple cases for the energy shift.

- **A** : the dispersion is **identical** for both beams ($D_{u1} = D_u = D_{u2}$). There is **no** shift in the CM energy.

	IP2	IP4	IP6	IP8
$< D_y^* >$ (mm)	2.00	-1.25	1.85	-1.10
4 Bunches/Train : a,b,c,d				
y_{max} (μm)	7.2	6.1	6.5	6.4
δE_{max} (MeV) [$\sigma_y^* = 5\mu$m]	44.6	25.4	37.4	24.2
δE_{max} (MeV) [$\sigma_y^* = 7\mu$m]	24.4	13.4	20.4	12.6
3 Bunches/Train : a,b,c				
y_{max} (μm)	4.0	4.1	4.1	4.1
δE_{max} (MeV) [$\sigma_y^* = 5\mu$m]	24.8	17.0	23.6	15.6
δE_{max} (MeV) [$\sigma_y^* = 7\mu$m]	13.6	9.0	12.8	8.0
2 Bunches/Train : a,b				
y_{max} (μm)	1.6	1.3	1.3	1.6
δE_{max} (MeV) [$\sigma_y^* = 5\mu$m]	10.0	5.4	7.4	6.0
δE_{max} (MeV) [$\sigma_y^* = 7\mu$m]	5.4	2.8	4.0	3.2

Table 1: Comparison of the average vertical dispersion $< D_y^* >$, the maximum separation of the bunches in the train y_{max} and the corresponding maximum difference in CM energy δE_{max} between the collision points. The difference in CM energy is calculated for values σ_y^* of 5 and 7 μm. The separation of the bunches decreases monotonically with the number of bunches per train. The current per bunch is 500 μA in all cases. These numbers have been generated with the l05p46v3 optics.

- **B** : the dispersions have the **opposite sign** ($D_{u1} = D_u = -D_{u2}$). The shift of the mean CM energy, due to the collision offset, is given by :

$$\Delta E_{CM} = -2u_0 \, \frac{\sigma_E^2 D_u}{E_0 \sigma_B^2} \tag{18}$$

  Using the same numerical values for ε_y, β_y^* and σ_ϵ that were used previously, we obtain a CM energy shift at 45 GeV of $\Delta E_{CM} = -3.3$ MeV for a separation of $2u_0 = 2y_0 = \sigma_B/4 \simeq 2\mu$m. This corresponds roughly to the typical accuracy of the adjustment of the collision offsets with the vertical separators. With constant beam size, the luminosity does not vary by more than $\pm 1.5\%$ when y_0 is varied between $\pm\sigma_B/4$. Such a CM energy shift is significant since the 1995 energy scan aims at accuracies below 1 MeV.

4 Consequences for the 1995 Energy Scan

The aim of the 1995 energy scan is to reduce the contributions to the errors on M_Z and Γ_Z due to the beam energy calibration to about 1 MeV. To reach such a level, all systematic effects or corrections must be controlled at the level of $\sim 10^{-5}$.

Operation of LEP with bunch trains leads to vertical dispersion at the IPs of up to 2.0 mm [2, 4], which may be of opposite sign for the two beams. Simulations of LEP with trains of 2,3, and 4 bunches have been performed for the **l05p46v3** optics by E. Keil [4]. With bunch currents of 500 μA, the vertical position offset of bunches in a train reaches $y_{max} = 7.2\mu$m between the extreme bunches. The largest separation between colliding bunches is then close to $\approx \sigma_B$. The maximum difference in CM energy between the different collisions is given by :

$$\delta E_{max} = -2y_{max} \, \frac{\sigma_E^2 D_y}{E_0 \sigma_B^2} \tag{19}$$

	IP2	IP4	IP6	IP8
y_{max} (μm) [$I_b = 500\mu$A]	7.2	6.1	6.5	6.4
y_{max} (μm) [$I_b = 350\mu$A]	5.0	4.3	5.2	4.4
y_{max} (μm) [$I_b = 200\mu$A]	2.8	2.4	2.9	2.3

Table 2: Comparison of the maximum separation of the bunches in the train y_{max} for the case of trains with 4 bunches as a function of the current per bunch I_b. As expected the separation scales roughly with the current. All numbers have been provided by A. Verdier.

In table 1 the maximum CM energy difference between the collisions is shown as a function of the number of bunches in the train for vertical beam sizes at the IPs of 5 and 7 μm. The values span the ranges between 3 and almost 50 MeV. The dispersion varies from one IP to another because of the RF sections around IPs 2 and 6 which impose additional constraints on the optical functions. The bunch to bunch dispersion difference reaches about about $\pm$ 0.1 mm in trains of four bunches and vanishes for trains of 2 bunches. Simulations show that machine imperfections should not affect the difference of dispersion between bunches in a train. These differences are due to the beam-beam interaction. The combination of bunches a and d seem to be the best configuration for trains of 2 bunches since the average offset at 0.5 mA bunch current is reduced to slightly less than 1 μm [7]. Table 2 shows the evolution of y_{max} as a function of the bunch current. Since the separation is due to the beam-beam kicks, it scales with the bunch current.

4.1 Luminosity Optimization

When two bunches collide with an offset $2y_k$, where y_k can be positive or negative, the luminosity is reduced by a factor

$$\exp\left[-\frac{y_k^2}{\sigma_{yk}^2 + D_{yk}^2\sigma_\epsilon^2}\right] \tag{20}$$

Since the offsets vary along the train, the beams must be steered one against the other to an optimum position y_{opt}

$$\mathcal{L} = \sum_k \mathcal{L}_k \exp\left[-\frac{(y_k - y_{opt})^2}{\sigma_{yk}^2 + D_{yk}^2\sigma_\epsilon^2}\right] = maximum \tag{21}$$

or

$$\frac{d\mathcal{L}}{dy_{opt}} = 0 = 2\sum_k \frac{(y_k - y_{opt})}{\sigma_{yk}^2 + D_{yk}^2\sigma_\epsilon^2}\mathcal{L}_k \exp\left[-\frac{(y_k - y_{opt})^2}{\sigma_{yk}^2 + D_{yk}^2\sigma_\epsilon^2}\right] \tag{22}$$

in order to achieve the best luminosity performance. k labels the different collisions of the trains. $\mathcal{L}_k$ is the luminosity for collision k which is a function of the individual bunch currents and beam sizes. To ensure that the CM energy shift averages to zero in the same IP, we have to require that the luminosity weighted energy shift cancels :

$$< \Delta E_{CM} >_{IP} = 0 = -\frac{2\sigma_E^2}{E_0\mathcal{L}}\sum_k D_{yk}\frac{(y_k - y_{opt})}{\sigma_{yk}^2 + D_{yk}^2\sigma_\epsilon^2}\mathcal{L}_k \exp\left[-\frac{(y_k - y_{opt})^2}{\sigma_{yk}^2 + D_{yk}^2\sigma_\epsilon^2}\right] \tag{23}$$

It is clear that unless the dispersion is the same for all bunches in the train, these two optimums will not coincide. Simulations show differences of about $\pm$ 0.1 mm between the dispersion of the different bunches. The best steering for high luminosity (equation 22) will lead to shifts in the average CM energy in one IP of the order of 0.2 MeV for trains of 4 bunches (differences in beam sizes between the bunches have not been taken into account for this estimate). Because the beam size σ_{yk} is affected by the beam-beam interaction while the beams are scanned one against each other with the vertical separators, it is not trivial to adjust the beams in order to satisfy equations 22 and 23.

All the parameters that play a role for the energy shift are difficult to measure and will not be known with high accuracy. The vertical dispersion and collisions offsets at the IP are very difficult to measure. One should therefore try to minimize and smear out the energy shifts. In daily operation it is the luminosity that will be maximized (equation 22). If the luminosity curves as a function of the separator setting are measured accurately, it is possible to make the best possible adjustment of the separation. It should be noted that a complete and clean scan of the separators at all IPs will probably take at least one hour. These scans cannot be performed in parallel for all IPs because a large separation in one IP affects the luminosity of the other IPs. Once the luminosity is optimized, systematic CM shifts will appear at the IPs if there are systematic differences between the dispersions of the bunches in a train. Since the tunes of the bunches in a train can differ, the bunches will not have the same lifetimes. In addition, beam energy variations due to tides, temperature, etc. will cause the beams to drift in opposite directions ($\delta y \simeq \pm 0.5 \mu$m). These effects require periodic readjustments of the separators in the coast.

It is clear from table 1 that the situation improves significantly for trains of less than 4 bunches since the trains are less distorted. This leads to smaller CM energy difference. It becomes easier to adjust the trains one against the other. With two bunches in a train the offsets between the collisions are of similar size than the probable accuracy of the setting of the separators. But even for these shorter trains, it is still necessary to avoid any systematic bias of y_{opt} larger than about 0.2 to 0.3 μm since the dispersion is still present at the IP. This tight constraint is set by the requested accuracy (< 1 MeV) on the Z mass and width for a new energy scan. The possibility of controlling the collision offsets to this level is currently being studied.

Some information on this effect could be obtained by the experiments provided the Z statistics could be high enough : the experiments might try to measure a systematic difference in the cross-section between the different collision points for off-peak fills. But it is not clear if this is feasible with the expected event statistics and systematic miss-crossings of the beams cannot be detected in this way. But such measurements might help to understand the dispersion at the IPs.

It is important to remember that with the present accuracy on the mass and width of the Z, the correction to the CM energy spread cannot be neglected and has also to be well understood. This obviously requires some knowledge of the vertical dispersion at the IPs. It also clear that even with well adjusted collisions of trains, the differences in CM energy add to to energy spread. Again, fewer bunches in a train reduces this effect.

For the 1993 energy scan, the dispersion measurements will be reanalyzed to evaluate limits on possible differences of vertical and horizontal dispersion at the IPs. Such differences might have been produced by the vertical separation at the uneven IPs or by coupling between the horizontal and vertical planes.

5 Consequences for LEP2

The beam energy spread σ_E increases proportionally to E^2 when the beam energy varies. Because of the limited strength of the vertical separators, the vertical separation bumps will be smaller at LEP2. This will reduce the vertical dispersion and increase the collision offsets. One can expect an increase of the CM energy shifts of about a factor 2 to 4 at LEP2 since the vertical beam sizes will not vary significantly. This is small enough compared with the expected W mass precision. However, since current experience with bunch trains is limited and the precise parameters that will be used for operation with bunch trains at LEP 2 are not final, this effect should be carefully watched.

6 Conclusion

Vertical dispersion and collision offsets at the IP leads to shifts of the average CM energy and to changes of the CM energy spread. The expected CM energy shifts could reach about 50 MeV between the different bunches in the case of trains of four bunches and small vertical beam sizes. The systematic errors will be difficult to control since all quantities that are needed to calculate these effects are not known with good accuracy. Since the situation becomes simpler with shorter trains, one should consider the possibility of performing the energy scan with only 2 or 3 bunches per train. Yet even with shorter trains, the presence of vertical dispersion requires very accurate and unbiased adjustments of the collisions.

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